

## Errors

### Chapter 1

- 1) page 17, def. 1.1.1: such that  $(R, \cdot)$  is a commutative monoid
- 2) page 18, after 1.1.3: For every ring  $R$ , there exists a unique ring homomorphism ....
- 3) page 32, proof of prop. 1.2.7, line -3: we get  $\beta_p \leq \min\{\alpha_{p1}, \dots, \alpha_{pm}\}$
- 4) page 39, tutorial 6.d, hint: ... and consider  $\varepsilon(\sum_{i=1}^r g_i a_i h_i + (f))$ .
- 5) page 43, prop. 1.3.5, end of proof: since the first components of  $v_1, v_2, \dots$  form a non-decreasing sequence.
- 6) page 45, cor. 1.3.10, proof, line 4: ... for every  $i \geq 2$ . Then we have  $\langle t_2, \dots, t_i \rangle \subseteq M_i$ , and therefore  $t_{i+1} \notin \langle t_2, \dots, t_i \rangle$ . Thus the monomial submodule  $\langle t_2, t_3, \dots \rangle$  ...
- 7) page 55, prop. 1.4.18, proof, line 3: if there is a non-empty subset  $\Sigma' \subseteq \Sigma$  having ...
- 8) page 77, def. 1.7.4, line 3: or simply a  $\Gamma$ -graded  $R$ -module if  $\Sigma = \Gamma$
- 9) page 80, prop. 1.7.12, proof, line -4: the sum extends only over  $(k, \ell) \neq (i, j)$
- 10) page 81, example 1.7.13, line -3: the cases  $f \in R_0$ ,  $g \in R_1$  and  $f \in R_1$ ,  $g \in R_0$  are missing

### Chapter 2

- 1) page 88, prop. 2.1.2, line 6: of  $P^r$ .
- 2) page 101, line -4, proof of prop. 2.3.3:  $(\sum_{j=1}^s c_j \text{LC}_\sigma(g_j))te_i \in (P^r)_{te_i}$ . Therefore  $\Lambda$  is ...
- 3) page 106, prop. 2.3.10, proof, line 5:  $\bar{m} \in \text{Syz}_P(\text{LM}_\sigma(\mathcal{G})) \setminus \{0\}$ ,
- 4) page 108, tutorial 19.c: in 1), 2), 3), 4), the symbols  $\subseteq$  should read  $\in$
- 5) page 117, proof of lemma 2.4.16, line 5:  $\text{Syz}(\text{LM}_\sigma(\mathcal{G})) = \dots$
- 6) page 123, example 2.5.4, line 4:  $S_{12} = y^2 g_1 - x g_2 = -y^4 + x z^3 \xrightarrow{g_3} 0$
- 7) page 123, example 2.5.4, line 5:  $x^3 z^3 - x y^2 z^3 \xrightarrow{g_2} 0$  should read  $x^3 z^3 - x y^2 z^3 \xrightarrow{g_1} 0$
- 8) page 123, theorem 2.5.5: The formulation of the theorem and its proof can be improved by constructing a tuple  $\mathcal{H}$  and using  $B$  instead of  $\mathbb{B}$ .
- 9) page 127, exercise 5.a: ... consists of binomials or monomials.
- 10) page 127, exercise 6.d: elements of  $P^3$
- 11) page 128, tutorial 23.b:  $\mathbb{Q}[x_1, x_2, x_3]$
- 12) page 130, tutorial 24.f: ... has (up to sign) the same reduced Gröbner basis
- 13) page 131, tutorial 25.e: such that  $t\sigma_{ij} + t'\sigma_{jk} - t''\sigma_{ik} = 0$ . Prove that one can choose  $t = 1$  if and only if ...

- 14) page 136, theorem 2.6.6, line -5 of the proof: before “By induction” insert: Hence  $B$  is isomorphic to  $A[x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n]/\mathfrak{n}$  for some maximal ideal  $\mathfrak{n}$ .
- 15) page 138, prop. 2.6.11, proof, line 4: Using the representation  $f = q_1(x_1 - a_1) + \dots + q_n(x_n - a_n) + p$  provided by Theorem 1.6.4, we then see that such a polynomial  $f$  belongs to  $\mathfrak{m}_p$ .
- 16) page 140, thm. 2.6.16, proof, line 5: we may assume that  $\mathcal{I}(\mathcal{Z}(I)) \neq (0)$ .
- 17) page 142, exercise 8a: Show that if  $I$  is irreducible then  $\mathcal{Z}(I)$  is irreducible.

### Chapter 3

- 1) page 149, definition 3.1.1: In the second volume, the definition is changed to  $ju \leq i$  in line 5.
- 2) page 150, prop. 3.1.4.a:  $\deg_{\sigma, G}(\sigma_{ij}) >_{\sigma} \text{LT}_{\sigma}(\lambda(\sigma_{ij})) = \max_{\sigma}\{\dots\}$
- 3) page 151, line -8, example 3.1.7:  $S_{34} = \dots = x_1x_3^2g_2 + x_2^2x_3^2g_1 - x_2^3g_4$
- 4) page 151, line -6, example 3.1.7: the last column of the matrix should be  $(-x_2^2x_3^2, -x_1x_3^2, x_2x_4^2, -x_1^3x_3 + x_2^3)$
- 5) page 155, exercise 1, line 1: Show that the module ...
- 6) page 155, exercise 4: if and only if  $s = 1$  and  $g_1 \neq 0$ .
- 7) page 155, tutorial 28, line 2:  $\mathbb{R}^{k+1}$
- 8) page 158, tutorial 28.g.1: where  $i = 1, \dots, k-1$  and  $x_0 = x_k = 0$ .
- 9) page 158, tutorial 28.g.2, line 2:  $\gamma_i(x - c_{i-1}) + d_{i-1}$
- 10) page 159, tutorial 29.a: Show that the  $P$ -modules  $M$  and  $P^r/M$  are free.
- 11) page 162, lemma 3.2.2, line 2: be given by  $\lambda(\varepsilon_i) = g_i$
- 12) page 162, prop. 3.2.3, line 2: given by  $\lambda(\varepsilon_i) = g_i$
- 13) page 163, example 3.2.4, line 6:  $v_3 = (-x_2, x_3^2, 0, 0, -x_2)$
- 14) page 163, example 3.2.4, line 8:  $\dots, x_1x_3^2 - x_1^2, x_1x_2x_3$ .
- 15) page 170, example 3.2.21, line 1: When we apply the first part of this lemma ...
- 16) page 173, exercise 4: three  $R$ -submodules
- 17) page 173, exercise 6.b: the columns of  $\mathcal{A}$  are contained in the module ...
- 18) page 182, prop. 3.3.9.a: The  $P$ -linear map  $A_{r,s} : \dots$
- 19) page 186, theorem 3.3.15.c: by the residue class  $(a_{11}, a_{21}, \dots, a_{s1}, \dots, a_{1r}, a_{2r}, \dots, a_{sr}) + U$ . Then ...
- 20) page 194, tutorial 33.i, hint: and apply  $\text{Hom}_R(R/I, -)$  to the exact sequence ...
- 21) page 204, line -6, tutorial 35:  $(p_0, \dots, p_n) \in K^{n+1} \setminus \{0\}$
- 22) page 205, tutorial 35.i: delete the sentence “Let  $\{e_1, \dots, e_4\}$  be the canonical basis of  $K^4$ .”
- 23) page 210, tutorial 36.m: that whenever  $C(\alpha_1, \dots, \alpha_n) > C(\beta_1, \dots, \beta_n)$  ... we have the inequality  $x_1^{\alpha_1} \cdots x_n^{\alpha_n} >_{\hat{\sigma}} x_1^{\beta_1} \cdots x_n^{\beta_n}$ .

- 24) 216, example 3.5.10: replace  $\dots :_P (x_1)$  six times by  $\dots :_P (x_2)$
- 25) page 218, theorem 3.5.13, proof, line -3: we can use the lemma to get  
 $v \in NP[y] :_{P[y]^r} (f_1 \dots)^\infty$ .
- 26) page 218, example 3.5.14: replace  $I :_P (x_1)^\infty$  two times by  $I :_P (x_2)^\infty$
- 27) page 223, tutorial 38.h: delete the hint
- 28) page 238, tutorial 40.f, line 5: Assume that  $K[f_1, \dots, f_s] \subset P^G$  and choose a homogeneous polynomial  $g \in P^G \setminus K[f_1, \dots, f_s]$  of minimal degree.
- 29) page 243, prop. 3.7.1, proof, line 4: defined by  $x_i \mapsto a_i$  for
- 30) page 246, example 3.7.6, line 8: How sharp is this bound?
- 31) page 254, example 3.7.20, line -2:  $z^2 - 5z = 0$
- 32) page 256, thm. 3.7.23, proof, line -5:  $\dots = (x_n - a_i)$  by construction ...
- 33) page 257, proof of thm. 3.7.25, line -3:  $\{\text{LT}_{\text{Lex}}(x_1 - g_1), \dots, \text{LT}_{\text{Lex}}(x_{n-1} - g_{n-1}), \text{LT}_{\text{Lex}}(g_n)\}$
- 34) page 259, line -3:  $g_3 = (x_3 - 2)(x_3 + 3)(x_3^2 - x_3 - 8)$
- 35) page 260, example 3.7.27, line -4:  $h_4 = y_3^3 + 2y_3^2 - 11y_3 - 24 = 0$
- 36) page 262, tutorial 42.g:  $\ell(f^2)$

## Appendices

- 1) page 306, exercise 1.3.6: Therefore  $\{b_{i_1}, \dots, b_{i_s}\}$  generates  $\Delta$ .
- 2) page 306, exercise 1.3.9:  $\dots = \cup_{i=1}^r \mathbb{T}^n e_i$
- 3) page 308, exercise 3.6.4: for  $i = 1, \dots, s$
- 4) page 309, section title “Special Sets” should be smaller
- 5) page 309, subsection 1, line -2: projective space associated to  $K^{n+1}$
- 6) page 311, last line of subsection 4:  $\det(\frac{\partial f_i}{\partial x_j})$